Modelling of quantum electron transport in high magnetic fields and the application to scanning gate microscopy (SGM)

The actual research activities concerning modelling of quantum electron transport in high magnetic fields are the result of our long term research activities that started with epitaxial growth and experimental magneto transport investigations of 3D and quasi 2D electron systems (wide quantum wells and doping super lattices) based on narrow gap semiconductors. A major goal from the very beginning was the investigation of the magnetic field induced metal insulator transition and Wigner condensation of electrons, which is also an important ingredient of the quantum Hall effect in 2D electronic systems. In this context the physics of the quantum Hall regime was a continuous driving force while shifting the research activities more and more from experiments to theory and simulation. The unique electronic regime of wide quantum wells, which can be understood as an intermediate regime between 2D and 3D, demanded novel theoretical approaches to electron transport since neither 3D physics nor well defined 2D physics could be applied to our self-grown wide quantum wells.

Besides the quantum Hall effect (QHE) itself, also magneto resistance fluctuations, magneto conductance fluctuations and so called edge channel transport are important indications of the quantum nature of magneto transport in the so called QHE regime. A very fundamental challenge of any theoretical approach to these effects is the fact, that the underlying electron systems consist of a many particle quantum state, while the transport for investigating these electron systems is driven by non- equilibrium, that is introduced by the contacts. Since stationary quantum states do not directly provide transport, different methods have been developed by different schools in order to overcome this counterintuitive requirement.

We have been able to contribute another new approach to non-equilibrium transport that provides also the possibility to combine it with nearly every model for the steady state of the underlying electron systems. A detailed presentation and discussion of our model in contrast to other so far existing models can be found in [1]. We started a cooperation with the group of R. Römer at Warwick University, who are experts for the calculation of stationary many particle quantum states and made plans for joining their Hartree-Fock (HF) model and our non-equilibrium network model (NNM) for transport. The idea to join these models has already been announced in an early common paper [2]. However, no clear concept had been at hand for this purpose at that time, which took several more years. Meanwhile the non-equilibrium network model has been successfully used on the basis of a semi classical approach to the microscopic nature of the electron system, as can be seen in Ref. 3

Quite a number of extensions and modifications had to be made for achieving the present status of our simulation model. That was necessary not only for handling the data transfer between the HF model and the NNM, but also in order to account for the physical boundary conditions as given under real experimental conditions. By extensive testing in the last two years it turned out that mainly two adaptations had to be made for the HF model: (i) Introducing a smooth Fermi edge in order to account for finite temperatures for the occupation of the quantum states; (ii) Introducing a predefined Fermi energy into the HF procedure instead of fixing the number of electrons in the electronic system, as done before. This is necessary because in real systems there always exists a 2D electron reservoir, in which the investigated structure is embedded. Such a reservoir pins the magnetic field dependent Fermi energy and leads to the effect that electrons will move in and out of the device under investigation while changing e.g. the magnetic field. Consequently also a module had to be added to the HF model, which calculates the 2D Fermi level as a function of magnetic field at given 2D carrier density for the host electron system. These improvements have finally led to more realistic results, but also improved the convergence behaviour of the self-consisted overall procedure.

Simulation of scanning gate microscopy (SGM)

One major goal at present is the application of our model to SGM, which has the potential of non-invasive probing of quantum states that could become interesting in context with quantum computing. In this context also SGM combines the counterintuitive requirements of investigating stationary quantum states by using non-equilibrium electron transport. In the following we show briefly some recent preliminary results that demonstrate the application of SGM for real space imaging of condensed many particle quantum states.

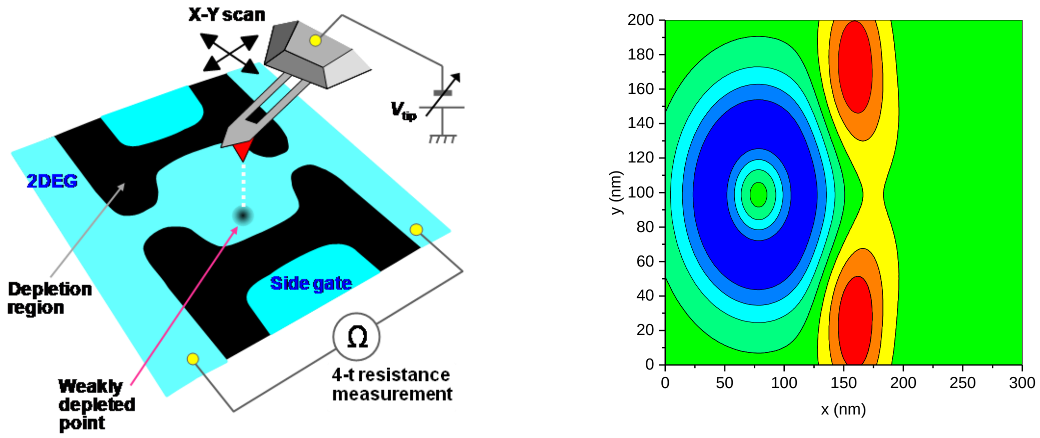

FIG.1. Left: Scheme of SGM, where a biased tip scans close to the surface and affects the buried electron system. The conductive response of the whole device is recorded as a function of tip position. **Right:** bare electrostatic potential of the simulated model system that consists of a ring shaped quantum dot nearby a quantum point contact. This structure has to be inserted between the electrodes on the left. Red means high and blue low potential for electrons that consequently will collect in the (blue) ring.

In Fig.2 on the left it can be seen, that the response pattern of the total sample current reflects the geometry for the case of a condensed quantum state (Wigner crystal) shown on the right. The non-locality of the quantum state is evident from the fact, that the contrast does not depend on the distance of the tip from the QPC, where the tip hits the quantum state. The calculations have been made according to the material parameters of a GaAs System.

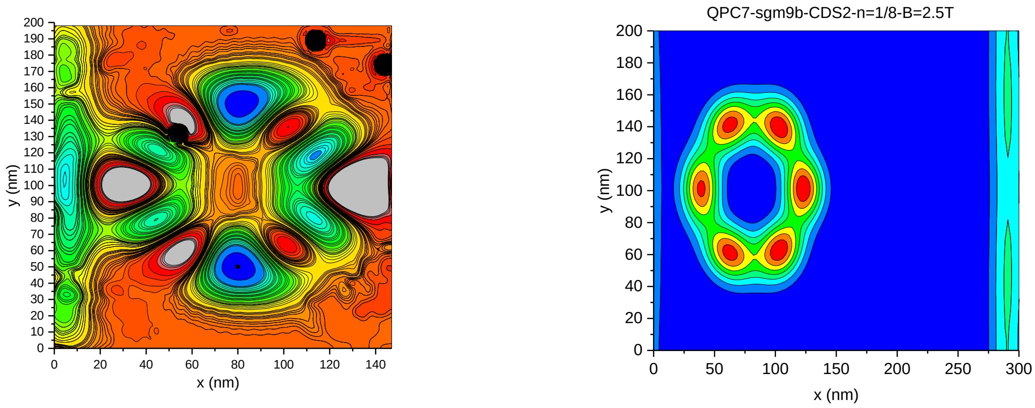

Fig.2 Left: Simulated conductive response (in arbitrary units) as a function of tip position (5mV tip potential) for the two-dimensional potential on the right of Fig. 1 that contains 8 electrons (note the different scale of the x-axis). **Right:** charge density of the condensed 8-electron many particle quantum state in the ring, that is obtained by the Hartree Fock model for the above ring potential.

[1] J. Oswald, M. Oswald, J. Phys. C **18 – 7,** R101-R138 (2006)
[2] C. Sohrmann, J. Oswald, R.A. Römer R.A. : *Lecture Notes in Physics* **762**, pp. 163-193 (2009)
[3] C. Uiberacker, C. Stecher, J. Oswald, Phys. Rev. B **86**, 045304 (2012)