Gate Controlled Narrowing of the Quantum Hall Effect Plateau Transitions

Josef Oswald, Christoph Uiberacker

Institute for Physics, University of Leoben, Franz Josef Str. 18, A-8700 Leoben, Austria

E-mail: josef.oswald@unileoben.ac.at

Abstract. The plateau transitions of the integer quantum Hall effect (QHE) are accompanied by dissipative bulk current flow, which causes the corresponding peaks in the longitudinal magneto-resistance R_{xx}. On the basis of a non-equilibrium network model for magneto-transport, we propose an experimental setup for suppressing the dissipation in the plateau transition regime. This is achieved by an appropriate biasing of a narrow stripe shaped gate electrode, which is aligned to the longitudinal direction of the sample and needs to cover only a vanishing small part (few percent) of the bulk region. Our calculations demonstrate, that the reduction of the dissipation is accompanied by a drastic narrowing of the plateau transitions. Besides the practical aspect in context with metrology, such an investigation provides additional insight into the role of quantum phase transitions and phase coherence for the appearance of the QHE.

1. Introduction

It is well accepted, that quantum localization in context with scaling theory plays a decisive role for the integer quantum Hall effect (IQHE)[1]. Nevertheless, the fundamental question about the mechanism of current flow in the macroscopic samples is still a challenging problem. If the quantum states in the bulk are localized on a length scale less than the sample size, the bulk of the sample is insulating and only states near the sample edges, so called edge channels (EC), remain de-localized. These ECs enable dissipation-less current flow, which causes a zero longitudinal resistance R_{xx}. By interpreting states in QHE samples as perimeters of clusters undergoing site percolation it was shown [2] that for zero temperature the localization length decreases like $\xi(E) \sim |E - E_c|^\nu$, with E_c the smallest energy for which an infinite cluster exists. Classical percolation gives $\nu = 4/3$ whereas $\nu = 7/3$ was proposed for the quantum case. Furthermore, the fraction of states that extend beyond R falls polynomially like $f(R) \sim R^{-41/48}$. For macroscopic samples the fraction of de-localized bulk states is therefore predicted by scaling theory to go to zero. The dissipation is monitored by non-zero longitudinal resistance ($R_{xx} > 0$) forming peaks in the plateau transition regime. Systematic experimental studies of the R_{xx}-peak width as a function of temperature (see [3, 4] and references therein) and frequency (see [5] and references therein) are theoretically described by the scaling theory of the IQHE (see e.g.[1]). Due to this theory, the R_{xx}-peak width should shrink to zero while approaching absolute zero temperature and letting the sample size go to infinity.

Note in this respect that the fraction of de-localized states becomes finite in the presence of a nonzero electric Hall field. It was estimated [2] that the fraction of states of infinite extend
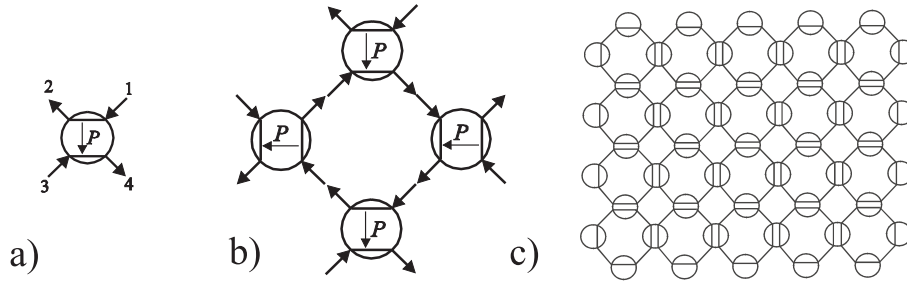
Figure 1. a) Node of the network with two incoming and two outgoing channels. The channels $1 \to 2$ and $3 \to 4$ are treated like ECs with back scattering, where $P = R/T$ corresponds to the relation between reflection and transmission coefficients according to the Landauer- Büttiker formalism. b) Arrangement of the nodes for building the minimal physical element of a network, which is the closed loop of a so called magnetic bound state. c) The complete network is composed by putting together a sufficient number of such adjacent loops.

goes like $f(\mathcal{E}) \geq e\mathcal{E}/(e\mathcal{E} + M)$ with the modulus of the field $\mathcal{E}$, e denotes the elementary charge and we defined $M := |\nabla V(x,y)|$ with $V(x,y)$ the underlying slowly varying potential.

Temperature has two effects on the sample. First, the phase coherence length of the electronic states decreases with increasing temperature and consequently states become more de-localized. Second, the energy smearing around the Fermi energy should open an energy window of de-localized states. As a result both effects lead to a broadening of the R_{xx}-peaks with increasing temperature.

We raise the question, whether quantum localization is a principal requirement for the existence of the IQHE, or scaling is just a phenomenon of low enough T and $\mathcal{E}$. We argue that dissipation might be the essential quantity. We use a non-equilibrium network model for the IQHE regime to demonstrate, that it is sufficient to control dissipation in order to control the sharpness of Hall plateau transitions without explicitly manipulating the phase coherence length.

2. The non-equilibrium network model

The layout of our network looks similar to the well known Chalker-Coddington (CC) network [6]. However, the CC network has been set up to study quantum localization, while our model aims directly at modeling non-equilibrium potentials and currents. Consequently our handling of the nodes as well as the association of the channels with currents and potentials is substantially different from the CC model. The main facts are given below and for further details refer to the cited papers[7, 8, 9]. Fig.1a shows a single node of our network, which transmits potentials from the incoming to the outgoing channels, Figs.1b-c demonstrate, how the network is composed. The potentials transmitted by one node are calculated by using $u_2 = (u_1 + P \cdot u_3)/(1 + P)$ and $u_4 = (u_3 + P \cdot u_1)/(1 + P)$. In contrast to the CC-network, we attribute the non-equilibrium currents to channel pairs, like e.g. the current from the right to the left $I = (e^2/h) \cdot (u_1 - u_4)$ (see Fig.1a). It is important to realize, that in this way our nodes provide a handle to both, the injected currents and the potentials. P results from tunneling across saddle points of the potential landscape in the bulk[7]: $P = \exp\left[-\frac{L^2 E_F}{e\tilde{V}} \frac{eB}{h}\right]$, E_F is the Fermi energy relative to the saddle energy which corresponds to the center of the LL, eB/h is the number of LL states, L is the period and $\tilde{V}$ the amplitude of a representative two-dimensional Cosine-potential modulation, which has the same Taylor expansion like the actual saddle potential. Therefore the ratio $L^2/\tilde{V}$ can be understood as a measure of the "smoothness" of the potential modulation near the saddle.

The design of the sample is managed by shaping the lateral confining bare potential and using a self consistent Hartree type approximation to calculate the Fermi energy and the lateral carrier distribution. On this basis a gate electrode can be easily modeled by biasing the bare potential of the designated gate region.

3. Numerical Results and Discussion

For our numerical study we use the shape of a Hall bar, like schematically shown in the insert of Fig. 2 with an additional narrow stripe like gate electrode. The Hall conductor is represented by 165x20 grid periods, the voltage probes are 5 grid periods wide and 5 grid periods long and are 20 grid periods apart from each other, the gate stripe is 2 periods wide and the length is varied in 3 steps: 140, 36 and 16 grid periods, denoted as long, medium and short respectively. The bulk carrier density was chosen to be $n = 4x10^{11}cm^{-2}$. The key point of our concept is that one has to keep the gate potential tuned to the closest integer filling, because the gate region should get insulating without generating an edge channel at the boundaries.
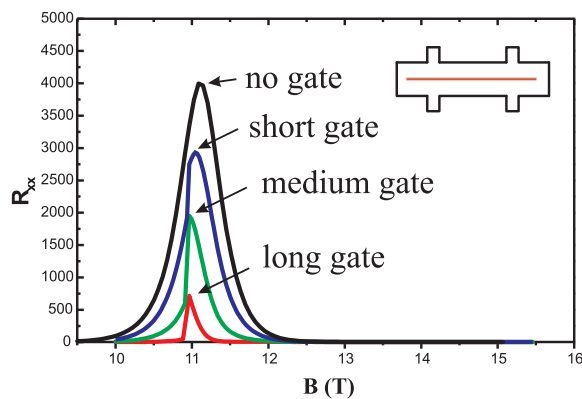
Figure 2. Simulated R_{xx} versus B at different gate stripe lengths. Insert: Layout of the sample with gate (bold line).
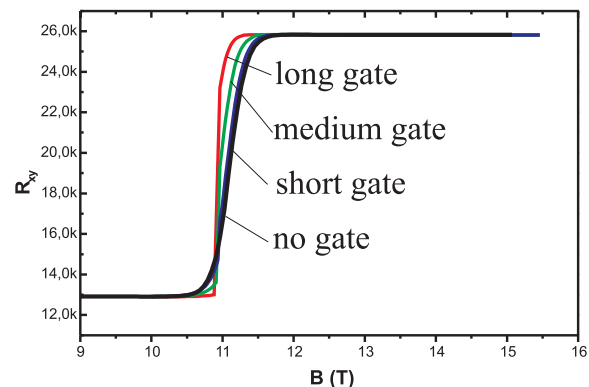
Figure 3. Simulated R_{xy} versus B at different gate stripe lengths

Figs.2 and 3. show the simulation results for the 3 different gate stripe lengths. One can clearly see, that the longer the gate is, the smaller is the R_{xx} - peak, which demonstrates, that the suppression of dissipation depends on the length of the gate. Looking at the Hall resistance, we observe at the same time a sharpening of the quantum Hall step. From the experimental point of view, the requirement of keeping the gate stripe at integer filling while sweeping the magnetic field might not be very practicable. Therefore we propose to measure at fixed magnetic field, but sweeping the gate voltage like shown in Fig.4: The magnetic field is kept fixed already before arriving at the next QHE plateau. Now we sweep the gate voltage towards depletion and at the right condition R_{xy} goes up the exact next plateau value, while R_{xx} drops to zero (not shown). Sweeping the gate potential further, R_{xy} departs from the plateau value again, demonstrating, that only a limited gate potential window exists for the proposed condition.

The narrow gate leaves almost all of the bulk states unaffected, which means that in case of a real sample also the phase coherence length of most of the bulk states would remain unchanged. The experimental observation of a non-zero longitudinal resistance requires dissipation in the bulk, which usually monitors the existence of de-localized states in the bulk. The physical picture of the gate action is that the gate introduces a narrow incompressible stripe, which divides the sample into two halves that act as extremely wide edge stripes without back scattering. In this case the existence of de-localized states in the bulk is no longer monitored by dissipation,
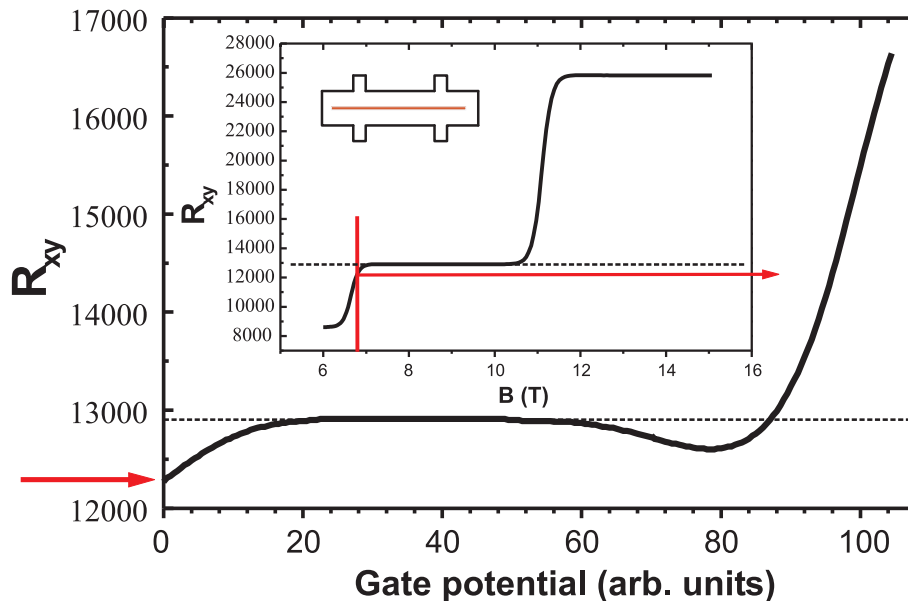
Figure 4. R_{xy} versus gate potential at a fixed magnetic field B. Insert: B-sweep of R_{xy} without gate. The vertical line marks the magnetic field for the gate sweep, the horizontal arrow indicates the starting value of R_{xy} for the gate sweep. The broken line marks the $\nu = 2$ plateau value.

which results in a zero longitudinal resistance even in the presence of de-localized bulk states. In other words, by dividing the compressible bulk into two separated wide compressible stripes the sample is effectively pushed into the plateau regime. As a consequence, quantum localization and quantum coherency on macroscopic length scales turns out to be not necessary for preserving the precise quantizing character of the edge stripes and the existence of the QHE as a whole.

4. Summary
We have presented a numerical study of the role of dissipation for the sharpness of the QHE plateau transitions. By using a narrow gate stripe in the bulk region, which is tuned to the nearest integer filling, we have demonstrated a possible suppression of dissipation, which is accompanied by a drastic sharpening of the Hall plateau steps. This demonstrates, that quantum localization is not necessarily the only mechanism behind the plateau transition width. Experimental verifications of this numerical results are encouraged.

5. Acknowledgments

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