

A New Model for the Transport Regime of the Integer Quantum Hall Effect: The Role of Bulk Transport in the Edge Channel Picture

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Abstract:

Based on a current balance between edge and bulk we obtain a modification of the Landauer-Büttiker formalism. The new aspect of this approach is an interpretation of edge channel (EC) backscattering in terms of a bulk current which couples the edges. This coupling is described by a novel backscattering parameter P , which is a function $P(\Delta\nu)$ of the Landau level filling. We show, that the most important features of transport can be modeled already without requesting a specific function for $P(\Delta\nu)$. In addition, a number of trends in R_{xx} and R_{xy} , from which most of them have been studied experimentally in the recent work of Shahar et al., can be reproduced by using a pure exponential function for $P(\Delta\nu)$.

Keywords: Two-dimensional Electronic System, Quantum Hall Effect, Edge Channels, Backscattering

1. Introduction:

Even more than 15 years after the discovery of the integer quantum Hall effect (IQHE) in two-dimensional electronic systems [1], the QHE is still one major topic of every conference on low dimensional electronic systems. Particularly the nature of the transitions between adjacent QH plateaus is still a controversial question and a quantitative modeling of the complete transport regime of the IQHE has not been given so far. While the quantized values of the Hall resistance are well described by the edge channel (EC) picture [2], it is widely believed that the EC-picture is insufficient to describe also the transport regime between the IQHE plateaus. However, non-local experiments for the IQHE [3] as well as for the fractional QHE [4] have

demonstrated, that in the transition regime between QH-plateaus EC- and bulk transport exist simultaneously. In this regime dissipative transport appears in the partly filled top Landau level (LL) while the contribution of the lower LLs is represented by dissipationless EC-transport. A similar picture in terms of a coexistence of edge and bulk transport has been used to explain the results of magnetotransport experiments in PbTe wide quantum wells. In that case the EC-system and the dissipative bulk system have been explained to result from electrons in different valleys of the many valley semiconductor PbTe [5].

It is widely accepted that in the plateau transition regime backscattering in the top LL enables dissipative bulk conduction if the Fermi level E_F is near the center of the broadened LL. In the bulk

region a transition to an insulating state occurs if E_F moves out of the center on either side [6]. In fact, this symmetry of the transport behavior with respect to the LL center makes ECs unlikely to be responsible for the characteristics of the inter plateau regime. The problem can be summarized as follows: By starting with E_F above a completely filled top LL, ECs are formed and the transport can be described by the EC-picture without backscattering. With E_F approaching the center of the top LL dissipation because of backscattering becomes possible and finally with E_F moving below the center of the top LL the associated pair of ECs disappears. Consequently the contribution of the top LL should be different if E_F is above or below the center. In contrast, the experimental results of the plateau transitions indicate a symmetric behavior of the top LL like one expects for pure bulk transport.

However, there can be found one particular regime, where an asymmetric transport behavior in a LL is also obtained in the experiments, namely the regime of the Hall insulator (HI): Using QHE samples with not too low disorder, the HI regime is entered directly from the $\nu = 1$ integer QHE regime without observing the fractional QHE and recent results [7,8,9] are presently stimulating also the ongoing studies of the inter-plateau transitions. An analysis of the transport ranging from the HI to the adjacent QH liquid regime suggests the existence of a close relation between the transport behavior in the two regimes [9]: By defining a critical filling factor ν_c it is possible to distinguish two regimes that are coupled by the relation $\rho_{xx}(\Delta\nu) = 1/\rho_{xx}(-\Delta\nu)$, where $\Delta\nu$ is the filling factor relative to ν_c . Another important experimental fact is the existence of a critical longitudinal resistivity ρ_{xx}^c , which appears at the transition point from the QH-liquid to the HI regime [7]. This critical point ν_c is indicated by the crossing of the temperature dependent ρ_{xx} traces and the value of ρ_{xx}^c was found to be close to h/e^2 . Using a tensor based analysis of the experimental data, Shahar et al. [10] have been able to extract the contribution of the top LL (referred to as ρ_{xx}^{top}) to the total ρ_{xx} in the transition regime between the 1st and 2nd QH-

plateau. They found that ρ_{xx}^{top} shows the same behavior like ρ_{xx}^{ins} in the HI regime, namely a monotonous increase with increasing magnetic field without any peak-like behavior. It was possible to collapse all temperature dependent traces onto each other by plotting ρ_{xx}^{ins} as well as ρ_{xx}^{top} with respect to $(\nu - \nu_c) \cdot T^{-\kappa}$ using the same $\kappa = 0.45$. Another experimental fact is that ρ_{xy} remains quantized on the $\nu = 1$ plateau also in the HI regime below the critical filling factor, while ρ_{xx}^{ins} already steeply rises. Furthermore Shahar et al. demonstrated, that the conductivity components in the HI regime (σ_{xx}^{ins} and σ_{xy}^{ins}) as well as the extracted components for the top LL (σ_{xx}^{top} and σ_{xy}^{top}) for the $1 \rightarrow 2$ plateau transition fulfill a semicircle relation ($\sigma_{xx}^2 + \sigma_{xy}^2 \propto \sigma_{xy}$). Such a semicircle relation has been interpreted as being the consequence of a close relation between σ_{xx} and σ_{xy} [11].

2. Modeling of backscattering:

Several attempts for modeling a four-terminal experiment with a discrete backscattering barrier or a disordered region between ideal conductors have been made already in the past [2,12,13]. As an example, for the case of a four-terminal arrangement Büttiker [2,12] obtains

$$R_{xx} = (h/e^2)[R/(N \cdot T)] \quad (1)$$

where N is the number of channels, R and T are the reflection and transmission coefficients of the barrier. The Landauer-Büttiker (LB) formalism is in principle general and the transmitting channels are not necessarily edge channels. However, for the explanation of the quantized Hall resistance values Büttiker uses the EC-picture and therefore the transmitting channels are ECs and N therefore has to be identified with the filling factor ν .

Fig.1 shows a typical configuration for studying the effect of EC-backscattering. By applying a gate voltage the 2 dimensional electron gas (2DEG) can be partly depleted, which allows to achieve a controllable reflection of ECs. An extensive

experimental study, in which a number of different gates are used, has been performed e.g. by Müller et al. [14]. All these cases had been well described within the framework of the LB formalism.

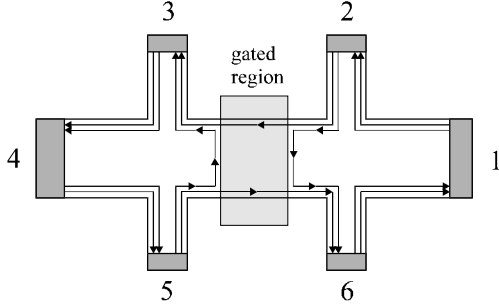


Fig.1: Schematic sample configuration for an experimental study of EC-backscattering. The reflection (backscattering) is forced by a gated region between ideal conductors. The ungated parts are assumed to be in the plateau regime, where there is no backscattering. The edge channels are indicated by the arrows.

However, there are some problems associated with the application of the LB formalism to the plateau transitions of the QHE: It describes the scattering region in terms of global transmission and reflection coefficients between ideal (scattering free) conductors. In addition, the LB formalism does not care about where the associated dissipation occurs. In the example of Fig.1 the additional dissipation due to the EC reflection occurs directly at the Hall contacts. In contrast, in the real situation of the QHE the whole sample enters the regime of finite ρ_{xx} at once and the dissipation occurs directly in the bulk implies automatically the existence of a dissipative current in the bulk, which must be related to the local bulk conductivity and hence to the particular bulk transport mechanism. The existence of such a dissipative bulk current is not addressed by the LB formalism. On this background the above referenced EC-approach together with the LB formalism leaves a missing link between the EC-picture and the local bulk transport properties and therefore it seems not to be applicable to the plateau transitions of the QHE.

In the following we are going to develop an alternative approach for EC-backscattering, which directly addresses the simultaneous presence of an edge current and a dissipative current in the bulk.

We consider a simplified situation in terms of an equivalent circuit, which allows us to present a short version of the more extended treatment presented in [15]. The situation can be understood to be obtained from Fig.1 by replacing the gate by an additional Hall contact pair. Dissipation can now be introduced by connecting a resistor across this additional contact pair.

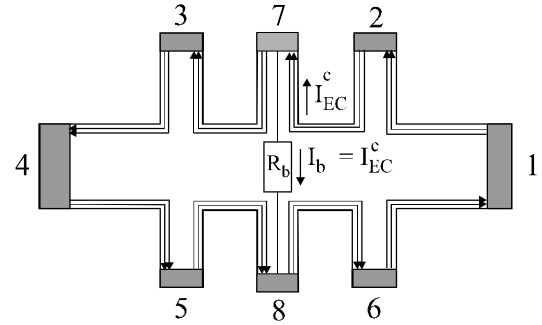


Fig.2: Schematic sample configuration for representing backscattering by a dissipative current across a resistor R , which connects both edges. As compared to Fig.1, the gate is replaced by an additional Hall contact pair with an ohmic resistor connected to it. Consequently the back scattering process is no longer represented by the reflection of ECs. The edge channels are indicated by the arrows.

The sample itself is supposed to be in the plateau regime, which means that without the artificial (bulk) resistor R_b there is no backscattering in the system, which means that $R_{xx} = 0$ between any longitudinal contact pairs at the same edge. The Hall voltage will be the same at all 3 Hall contact pairs, resulting in a Hall resistance of $R_{xy} = (1/\nu) \cdot (e^2/h)$, where ν is the number of EC-pairs. Now we are going to consider the effect of a dissipative current I_b across the contact pair 7-8 and calculate the resulting longitudinal resistance R_{xx} between the outer voltage probes 2 and 3. In order to preserve current conservation, the current I_b must be supplied through the contact arms by the ECs. This means that there must exist a potential difference (Hall voltage) between incoming and outgoing ECs in the contact arms of the middle Hall contacts 7 and 8. Since the incoming ECs carry the potential of the previous contact 2 and the potential of contact 7 is transmitted to contact 3 by the outgoing EC, the Hall voltage U_{xy}^c in the contact arm 7 must appear directly as the longitudinal voltage drop U_{xx} between

contact 2 and 3. The situation at the lower edge is similar and we obtain:

$$U_{xx} = U_{xy}^c = I_b \cdot R_{xy} = I \cdot R_{xx} \quad (2)$$

In this way we get a coupling between R_{xx} and R_{xy} by $R_{xx} = (I_b / I) \cdot R_{xy}$. We choose a dimensionless parameter $P = (I_b / I)$, which serves now as an alternative representation of backscattering as compared to the LB formalism:

$$R_{xx} = P \cdot R_{xy} \quad (3)$$

For the case of a 2D system R_{xy} has to be represented by $R_{xy} = h / (\nu \cdot e^2)$. The ratio of the currents $P = (I_b / I)$ can be interpreted as the ratio of the probability for an edge electron of being scattered to the opposite edge or not. At this point our approach formally meets the LB formula for $R_{xx} = (h / e^2)[R / (\nu \cdot T)]$ and a comparison yields the result that P corresponds formally to the ratio R / T of the LB formula. However, the main difference is that our result is directly obtained from considering the presence of a dissipative current which is not an edge current. In a more detailed treatment, which is presented in Ref.15, it is shown that the same result is obtained if one considers EC-backscattering to appear continuously all along the conductor like one has to expect in a realistic case and therefore we ask to refer to Ref.15 for more details. It is easily checked that Eqn. 3 also obeys energy conservation: The dissipation due to the sample current I is given by $I^2 \cdot R_{xx}$ which is the same as $I^2 \cdot P \cdot R_{xy} = I_b \cdot I \cdot R_{xy} = I_b \cdot U_{xy}$.

3. Modeling of transport:

For the case of a *single* LL, which is represented by a single pair of ECs, we substitute R_{xy} by h / e^2 and get

$$R_{xx} = \frac{h}{e^2} \cdot P \quad (4)$$

For a standard QH-system, as e.g. in AlGaAs/GaAs, backscattering appears only in the top LL in the

regime between plateaus, while the transport in the lower LLs remains dissipationless. For a transport model in the EC-picture one has therefore to combine one pair of ECs with non-zero backscattering ($P > 0$) and a set of EC-pairs without backscattering ($P = 0$). R_{xx} and R_{xy} of the complete system must finally result from the current distribution between both EC systems [16]. For treating these parallel systems we use the components of the conductance tensor G_{xx} and G_{xy} , which can be obtained from the components of the resistance tensor R_{xx} and R_{xy} by the well known relations $G_{xx,xy} = R_{xx,xy} / (R_{xx}^2 + R_{xy}^2)$. The use of these equations means that we restrict our analysis to the case of a symmetric behavior where $R_{xx} = R_{yy}$. In comparison to classical transport this corresponds to the case of a quadratically shaped conductor. Consequently the equations are formally identical with the equations for the resistivity ρ_{xx} , ρ_{xy} and conductivity σ_{xx} , σ_{xy} . In order to point out that the resulting quantities are not necessarily local quantities, we continue to use the symbols for global conductances $G_{xx,xy}$ and resistances $R_{xx,xy}$. Using Eqn.3 we get for the top LL:

$$G_{xx}^{top} = \frac{e^2}{h} \cdot \frac{P}{1 + P^2} \quad (5)$$

Due to the absence of backscattering in the lower LLs we have $G_{xx}^{low} = 0$ and therefore the total G_{xx} is given by G_{xx}^{top} . In an analogous way we calculate the Hall components:

$$G_{xy}^{top} = \frac{e^2}{h} \cdot \frac{1}{1 + P^2} \quad (6)$$

$$G_{xy}^{low} = \frac{e^2}{h} \cdot \bar{\nu} \quad (7)$$

where $\bar{\nu}$ is the number of filled LLs below the top LL. The total Hall conductance G_{xy} is given by the sum of Eqn. 6 and Eqn. 7.

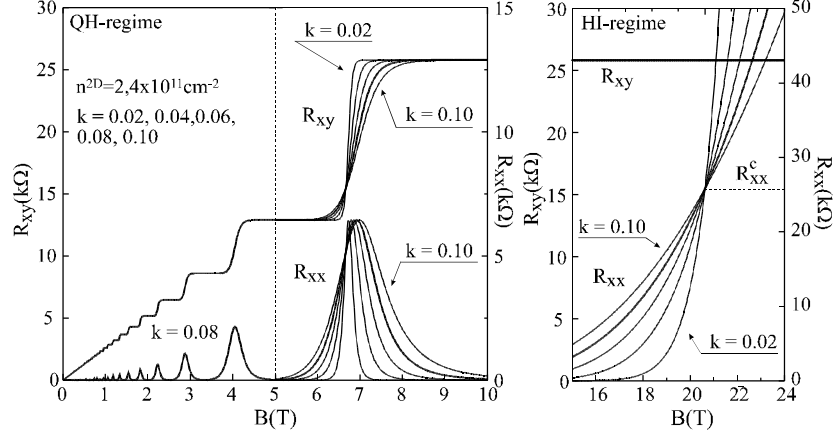


Fig.3: R_{xx} and R_{xy} calculated according to Eqn. 8, Eqn. 9 and Eqn. 10 for a sheet carrier density of $n^{2D} = 2.4 \cdot 10^{11} \text{ cm}^{-2}$ and different factors k in the exponent of $P(\Delta\nu)$. The range below $B = 5 \text{ T}$ shows just the traces for $k = 0.08$, the range above $B = 5 \text{ T}$ shows the traces for all different k values as given in the figure. The HI-regime is shown separately with a different R_{xx} scale on the right.

Now, using $R_{xx,xy} = G_{xx,xy} / (G_{xx}^2 + G_{xy}^2)$ we obtain Eqn.8 and 9:

$$R_{xx} = \frac{h}{e^2} \frac{P}{(\bar{\nu} + 1)^2 + (\bar{\nu} \cdot P)^2} \quad (8)$$

$$R_{xy} = \frac{h}{e^2} \left[\bar{\nu} + \frac{1}{1 + P^2} \right] \cdot \left[\bar{\nu}^2 + \frac{2\bar{\nu} + 1}{1 + P^2} \right]^{-1} \quad (9)$$

where P will be a function of the partial filling ν^{top} of the top LL. Even though we can obtain already quite a number of important results without knowing the specific function $P(\nu^{top})$, we will now continue to derive an appropriate function which is required for generating complete transport traces. A detailed discussion of results, which can be obtained already without knowing this function is given in the discussion section below.

Considering Eqn. 5 one can see that G_{xx} is proportional to P for $P \ll 1$, while it changes to a reciprocal dependence on P for $P \gg 1$. This implies that there exists a maximum of G_{xx} , which has to be identified with the experimentally obtained G_{xx} -peaks. If we now request a symmetric form of G_{xx} , we have to look for a suitable monotonous function $P(\nu^{top})$. To get perfect symmetry, the form of Eqn. 5 requires a function which fulfils the relation

$P(\Delta\nu) = 1 / P(-\Delta\nu)$ where $\Delta\nu$ is the filling factor of the top LL relative to the center. It is easily seen, that the only function which is also in agreement with the experimental observations [9] is of the form:

$$P(\Delta\nu) = \exp(-\Delta\nu / k) \quad (10)$$

with k being a constant but possibly temperature dependent factor $k(T)$. In order to get a curve without a point of inflection at $\Delta\nu = 0$, like experimentally observed, $\Delta\nu$ must appear linearly in the exponent. Since the maximum of G_{xx} is identified with the center of the top LL, $\Delta\nu$ is the filling factor relative to half filling. From Fig.3 one can see that the calculation based on Eqn. 8, Eqn. 9 and Eqn. 10 reproduces very well the typical traces known from the experimental curves at different temperatures.

4. Results and discussion

Although we have already given an appropriate function $P(\Delta\nu)$ which allows us to generate complete QHE traces, we will show that a number of fundamental results can be obtained already without needing a particular function for $P(\nu^{top})$. As a first

point one can directly see that Eqn. 5 and Eqn. 6 fulfill the semicircle relation $\sigma_{xx}^2 + \sigma_{xy}^2 \propto \sigma_{xy}$, which is valid also for the complete system. This semicircle relation was experimentally found to be valid for the top LL as well as for the HI-regime [10]. Based on the special form of Eqn. 5 we can distinguish two regimes which are divided by the point at which $P = 1$. In the following we will show that the regime $0 < P < 1$ can be attributed to the situation where the Fermi level E_F is located above the center of the top LL, while $P > 1$ corresponds to E_F below the center of the top LL.

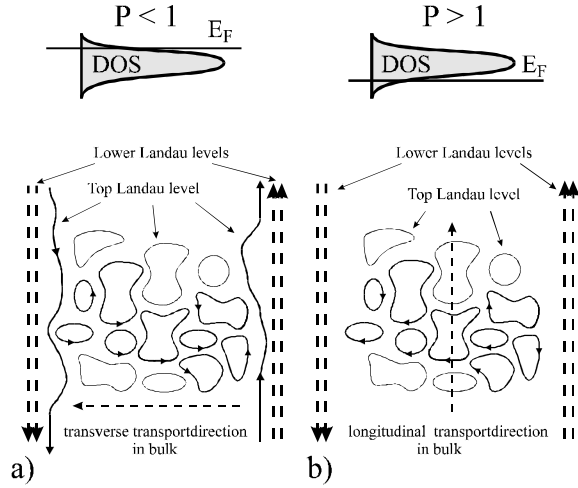


Fig.4: a) Edge channel conduction in the top-LL in the presence of localized magnetic boundstates. The transport across the loops appears as a transverse current, which acts as a backscattering process. b) Conduction in the top-LL in the presence of localized magnetic boundstates but in absence of an associated EC-pair. In contrast to the situation sketched in a), the transport across the loops appears now as a longitudinal current. The ECs of the lower LLs are indicated by the dashed arrows and are considered to be completely de-coupled from the top LL. The relative position of the Fermi level with respect to the LL is indicated at the top of the figure.

Fig.4a and Fig.4b show schematically the situation in the two regimes: While E_F is moving towards the center of the broadened top LL (Fig.4a), localized magnetic bound states are created in the bulk region in addition to the associated pair of ECs. Therefore some transport across those loops by e.g. tunneling becomes possible, which finally acts as EC-backscattering ($0 < P < 1$). According to Eqn.4, R_{xx}^{top} is directly proportional to the backscattering rate and hence proportional to the bulk conductivity in

this regime. For describing this type of transport in the bulk region, basically a network model such as e.g. that one of Chalker and Coddington [17] would be suitable. A situation with E_F below the LL center is schematically shown in Fig.4b with one major difference to Fig.4a, namely that the associated EC-pair is not present, while the transport mechanism in the bulk itself may remain the same like in the regime of Fig.4a. Consequently the transport in the bulk does no longer act as a coupling between opposite edges, but may contribute now via a current in the longitudinal direction instead. This will lead basically to a reciprocal dependence of R_{xx} on the bulk conductivity in the associated LL. One can interpret the two regimes as two different phases of the top LL with perpendicular directions of the dissipative bulk current. This is in striking agreement with Ruzin et al. [11] who also found, that for a correct description of the transport behavior the bulk current directions in both phases must be perpendicular to each other.

Characterizing the dissipative transport through the bulk by a conductivity σ_{bulk} , we get basically $R_{xx}^{top} \propto \sigma_{bulk}$ for E_F above the LL center and $R_{xx}^{top} \propto \sigma_{bulk}^{-1}$ for E_F below the LL center. Consequently any influence of an eventually existing temperature dependence of σ_{bulk} on the longitudinal transport properties must appear with opposite sign in the two regimes. This implies that there must be a crossover of the two regimes where the temperature dependence of R_{xx} is canceled. In this way our model indicates correctly the existence of metallic like and insulator like regimes. It is interesting to note that if one assumes an insulator like temperature dependence for the bulk conductivity σ_{bulk} in the whole transport regime, the temperature dependence of R_{xx} will appear metallic-like for $P < 1$ and insulator-like for $P > 1$, which is in agreement with the experimental observations. It is easily found that the critical (temperature invariant) point in the crossover regime occurs at $P = 1$. According to Eqn.4 this means that at the critical point R_{xx}^{top} approaches the quantized value h/e^2 . $P = 1$ also means that for the transport in a single LL $G_{xx} = G_{xy} = 0.5e^2/h$, in agreement with

Ref.18 . In [10] also the R_{xx} peak between the 1st and 2nd plateau has been analyzed. It has been found that the maximum value is $h/4e^2$ while R_{xx}^c at the critical point appears as $h/5e^2$. In our model the critical point appears at $P = 1$, for which we get a value of $R_{xx}^c = h/5e^2$, in agreement with [10]. Considering the maximum of Eqn.8 for $\bar{\nu} = 1$, we find $P = 2$, which leads to $R_{xx}^{\max} = h/4e^2$, also in agreement with [10].

Using the particular function $P(\Delta\nu)$ according to Eqn.10, we can go a step further: With the help of Eqn.4 we obtain $R_{xx}^{top} = (h/e^2) \cdot \exp(-\Delta\nu/k)$, which is a monotonous function and covers both regimes $P > 1$ and $P < 1$. Now we can also consider the principal behavior of σ_{bulk} in the tails of the LL ($P \gg 1$ and $P \ll 1$) by using $\sigma_{bulk} \propto R_{xx}^{top}$ for $\Delta\nu > 0$ and $\sigma_{bulk} \propto 1/R_{xx}^{top}$ for $\Delta\nu < 0$. As one expects for pure bulk transport, we obtain a symmetric function for σ_{bulk} around the LL-center $\sigma_{bulk} \propto \exp(-|\Delta\nu|/k)$. Thus it is demonstrated, that our model provides the correct framework to include also dissipative bulk transport.

The experimental evidence for the non-symmetric transport behavior of R_{xx}^{top} comes with E_F in the lowest LL ($\bar{\nu} = 0$, see Eqn.8). There R_{xx} is identical to R_{xx}^{top} and increases monotonously with decreasing filling factor. This is exactly the very well experimentally investigated regime of the HI: R_{xx}^{ins} has been indeed found to be monotonously increasing without any peak behavior and R_{xy} stays at the quantized value h/e^2 , in agreement with Eqn.9 for $\bar{\nu} = 0$. Therefore we can interpret the behavior in the HI regime to be a direct consequence of the asymmetric transport behavior of a single LL. Since in our model the transition to the HI as well as the inter-plateau transitions are described by the same function $P(\Delta\nu)$, the experimentally observed equivalent behavior of R_{xx}^{top} and R_{xx}^{ins} [10] is an inherent property of our model.

The fact, that the temperature dependence disappears at a certain point, suggests that the

temperature T enters only the factor k in the exponent of Eqn.10. Moreover, $\Delta\nu = 0$ in Eqn.10 means that $P = 1$ and therefore $R_{xx}^c = h/e^2$ for $\bar{\nu} = 0$, in agreement with [7]. This is also evident from Fig.3, where the traces cross each other at $R_{xx}^c = h/e^2$ (at $B = 20T$).

A widely used basis for the discussion of experimental data is the plot of the ρ_{xx} peak width ΔB as a function of temperature. In this context we analyze the width of the G_{xx} peak, which is described by Eqn.5: $G_{xx} \propto 1/(P + 1/P)$ is symmetric in P with respect to $P = 1$ and the maximum appears at $P = 1$. On the basis of this symmetry we choose a point on each side of the G_{xx} maximum. The associated values of the backscattering function $P(\Delta\nu)$ are P_1 and $P_2 = 1/P_1$, with P_1 being a constant, except unity. We can write $P_1 = \exp(\Delta\nu_1/k)$ and $P_2 = \exp(-\Delta\nu_1/k)$ and obtain an invariant expression by considering the relation $P_1/P_2 = P_1^2 = \exp(2\Delta\nu_1/k)$, where $2\Delta\nu_1$ can be identified as the width of the G_{xx} peak on the filling factor scale. Applying the logarithm results in $\ln(P_1^2) = 2\Delta\nu_1/k = const.$, which means that the temperature dependence of $2\Delta\nu_1(T)$ and $k(T)$ must be the same, regardless any particular form of $k(T)$. One gets temperature independent traces if one plots all temperature dependent traces for R_{xx} versus $\Delta\nu/k(T)$. In this way such ‘scaling plots’ of the whole R_{xx} can also be used to obtain the unknown function $k(T)$ from the experimental data. This was done by Shahar et al. and the fact that all experimentally obtained traces of R_{xx}^{top} and R_{xx}^{ins} of Ref.10 collapse onto a single trace, if plotted with respect to $(\nu - \nu_c) \cdot T^{-\kappa}$, suggests that the argument of the exponential function should have the form $\alpha \cdot (\nu - \nu_c) \cdot T^{-\kappa}$ with α being a constant. However, as evident from above, also any alternative temperature dependence $k(T)$ can be used in our model, such as e.g. $k(T) = \alpha + \beta \cdot T$, which has been suggested recently by Shahar et al [19]. Shahar et al.

have interpreted the appearance of this new temperature dependence as the indication of a new transport regime for the QHE. Since we have shown that many of the features can be modeled without needing a special form of the backscattering function, it might indeed happen, that different dominating bulk transport mechanisms cause similar experimental features.

5. Summary and conclusion:

In summary we have presented a model for the IQHE which makes use of a novel representation of backscattering. It successfully describes the full transport regime between the plateaus as well as the transitions between the QH-liquid and insulator regimes. Even though we use the edge channel approach for the IQHE as an input for our model, the results are more general and the model provides the correct framework to include also dissipative bulk transport. We have shown that already the interplay between edge and bulk leads to a number of important features, which seem to be insensitive to the nature of a particular bulk transport mechanism. Quite a number of well known facts can be obtained without needing the particular function of backscattering versus Landau level filling $P(\Delta\nu)$:

- (i) the semicircle relation between σ_{xx} and σ_{xy} for the complete QH regime as well as for the Hall insulator (HI) regime,
- (ii) the critical value $\rho_{xx}^c = h/e^2$ in the HI regime,
- (iii) the value $\sigma_{xx} = \sigma_{xy} = 0.5e^2/h$ at the critical point for a single Landau level,
- (iv) the maximum value $\rho_{xx}^{\max} = h/4e^2$ for the $1 \rightarrow 2$ transition,
- (v) the critical value $\rho_{xx}^c = h/5e^2$ for the $1 \rightarrow 2$ transition. Using an exponential function for $P(\Delta\nu)$ we obtain further:
- (vi) the validity of the relation $\rho_{xx}(\Delta\nu) = 1/\rho_{xx}(-\Delta\nu)$ between the HI and the adjacent QH-liquid regime,
- (vii) the equivalence of the temperature scaling of ρ_{xx} in the HI regime, of ρ_{xx} of the top LL and of the ρ_{xx} -peak width.
- (viii) regarding the temperature dependence (using any $k(T)$ monotonously increasing with T), the model indicates

correctly the existence of metallic like ($P < 1$) and insulator like ($P > 1$) regimes.

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- [16] In what follows we present an analytical version of our model. The results are fully in agreement with those of a numerical model which does not use the tensor relations. In the numerical model the sample current is allowed to flow via the two parallel EC systems, which are connected at the metallic contacts only. The potential differences at the contacts are then obtained in an iterative way from current conservation considerations. The numerical version of our model is able to give correct results for non-local contact configurations as well (considered for publication elsewhere).
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